ROMANIAN MATHEMATICAL MAGAZINE

If $a,b>0$ and $a^2+b^2=2$ then prove that:

$$\frac{a^3}{a+1}+\frac{b^3}{b+1}\ge 1$$

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Solution by Adgozel Adgozalov-Azerbaijan

$$\frac{a^3}{a+1}+\frac{b^3}{b+1}=\frac{a^4}{a^2+a}+\frac{b^4}{b^2+b}\ge\frac{(a^2+b^2)^2}{a^2+b^2+a+b}=$$

$$=\frac{2^2}{2+a+b}=\frac{4}{2+a+b}\ge\frac{4}{2+2}=1$$

$$\boldsymbol{Note}:$$

$$a^2+b^2\ge\frac{(a+b)^2}{2}\Rightarrow 2\ge\frac{(a+b)^2}{2}\Rightarrow (a+b)^2\le 4\ \Rightarrow a+b\le 2$$

$$''='' \text{ if } a=b=1$$