

ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$ and $ab + bc + ac = 3$ then prove that:

$$\frac{(2a + b + c)^3}{b + c} + \frac{(2b + a + c)^3}{a + c} + \frac{(2c + b + a)^3}{b + a} \geq 96$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\begin{aligned} & \frac{(2a + b + c)^3}{b + c} + \frac{(2b + a + c)^3}{a + c} + \frac{(2c + b + a)^3}{b + a} \stackrel{HOLDER}{\geq} \\ & = \frac{(4(a + b + c))^3}{3 \cdot 2(a + b + c)} = \frac{32(a + b + c)^2}{3} \geq \frac{32 \cdot 3(ab + bc + ac)}{3} = 96 \end{aligned}$$

Equality holds if $a = b = c$.