ROMANIAN MATHEMATICAL MAGAZINE

$$\text{If } x,y>0 \text{ and } x^2+y^2=2 \text{ then:}$$

$$\frac{1}{\sqrt{2x^2+5x+2}}+\frac{1}{\sqrt{2y^2+5y+2}}\ge\frac{2}{3}$$

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Solution by Tapas Das-India

$$2=x^2+y^2=\frac{x^2}{1}+\frac{y^2}{1}\overset{Radon}{\ge}\frac{(x+y)^2}{2}\ or,4\ge(x+y)^2\ or,x+y\le 2\ (1)$$

$$\frac{1}{\sqrt{2x^2+5x+2}}+\frac{1}{\sqrt{2y^2+5y+2}}=\frac{1}{\sqrt{(x+2)(2x+1)}}+\frac{1}{\sqrt{(y+2)(2y+1)}}\overset{AM-GM}{\ge}$$

$$\ge\frac{2}{(x+2)+(2x+1)}+\frac{2}{(y+2)+(2y+1)}=\frac{2}{3}\left(\frac{1}{x+1}+\frac{1}{y+1}\right)\ge$$

$$\overset{Bergstrom}{\ge}\frac{2}{3}\cdot\frac{(1+1)^2}{x+y+2}\overset{(1)}{\ge}\frac{2}{3}\cdot\frac{4}{2+2}=\frac{2}{3}$$

Equality holds for x=y=1