ROMANIAN MATHEMATICAL MAGAZINE

$$\text{If } x,y,z>0,\sum_{cyc}x=1 \text{ and } \lambda\geq 0 \text{ then :}$$

$$\sum_{cyc}\frac{x}{y(\lambda+y)}\geq\frac{9}{3\lambda+1}$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\sum_{cyc}\frac{x}{y(\lambda+y)}=\sum_{cyc}\frac{\left(\sqrt{\frac{x}{y}}\right)^2}{\lambda+y}\overset{\text{Bergstrom}}{\geq}\frac{\left(\sum_{cyc}\sqrt{\frac{x}{y}}\right)^2}{3\lambda+\sum_{cyc}x}\overset{\text{AM-GM}}{\geq}\frac{\left(3.\sqrt[3]{\sqrt{\frac{x}{y}}.\sqrt{\frac{y}{z}}.\sqrt{\frac{z}{x}}}\right)^2}{3\lambda+1}$$

$$\left(\because\sum_{cyc}x=1\right)=\frac{9}{3\lambda+1}\text{ and so, }\sum_{cyc}\frac{x}{y(\lambda+y)}\geq\frac{9}{3\lambda+1}\ \forall\ x,y,z>0,\sum_{cyc}x=1,$$

$$''=''\text{ iff }x=y=z=\frac{1}{3}\ (\text{QED})$$