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If $a,b,c>0,\sum_{cyc} a=6$ and $\lambda\geq 1$ then :

$$\sum_{cyc}\frac{a^2}{\lambda bc+\sqrt{a^3+8}}\geq\frac{3}{\lambda+1}$$

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$$\sum_{cyc}\frac{a^2}{\lambda bc+\sqrt{a^3+8}}\overset{\text{Bergstrom}}{\geq}\frac{\left(\sum_{cyc}a\right)^2}{\lambda\sum_{cyc}bc+\sum_{cyc}\sqrt{(a+2)(a^2-2a+4)}}$$

$$\overset{AM-GM}{\geq}\frac{\left(\sum_{cyc}a\right)^2}{\lambda\sum_{cyc}ab+\frac{\sum_{cyc}a^2-\sum_{cyc}a+18}{2}}\overset{\sum_{cyc}a=6}{=}\frac{2\left(\sum_{cyc}a\right)^2}{2\lambda\sum_{cyc}ab+\sum_{cyc}a^2-\frac{\left(\sum_{cyc}a\right)^2}{6}+\frac{\left(\sum_{cyc}a\right)^2}{2}}$$

$$=\frac{12\left(\sum_{cyc}a^2+2\sum_{cyc}ab\right)}{12\lambda\sum_{cyc}ab+6\sum_{cyc}a^2+2\left(\sum_{cyc}a^2+2\sum_{cyc}ab\right)}\overset{?}{\geq}\frac{3}{\lambda+1}$$

$$\Leftrightarrow 4(\lambda-1)\sum_{cyc}a^2\overset{?}{\geq}4(\lambda-1)\sum_{cyc}ab\Leftrightarrow 4(\lambda-1)\left(\sum_{cyc}a^2-\sum_{cyc}ab\right)\overset{?}{\geq}0\rightarrow\text{true}$$

$$\because\lambda\geq 1\text{ and so, }\sum_{cyc}\frac{a^2}{\lambda bc+\sqrt{a^3+8}}\geq\frac{3}{\lambda+1}\ \forall\ a,b,c>0\ \Big|\ \sum_{cyc}a=6\text{ and }\lambda\geq 1,$$

$$''=''\text{ iff }a=b=c=2\ (\text{QED})$$