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$$\text{If } a,b,c>0,\sum_{cyc} a^2=3 \text{ and } \lambda\geq 0 \text{ then :}$$

$$\sum_{cyc}\frac{a}{a+\lambda bc}\leq\frac{3}{1+\lambda abc}$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\sum_{cyc}\frac{a}{a+\lambda bc}=\sum_{cyc}\frac{a+\lambda bc-\lambda bc}{a+\lambda bc}=3-\lambda abc.\sum_{cyc}\frac{1}{a^2+\lambda abc}\overset{\text{Bergstrom}}{\leq}$$

$$3-\frac{9\lambda abc}{\sum_{cyc}a^2+3\lambda abc}\overset{\sum_{cyc}a^2=3}{=}3-\frac{9\lambda abc}{3+3\lambda abc}=\frac{3}{1+\lambda abc}\text{ and so,}$$

$$\sum_{cyc}\frac{a}{a+\lambda bc}\leq\frac{3}{1+\lambda abc}\ \forall a,b,c>0\ \Big|\ \sum_{cyc}a^2=3 \text{ and } \lambda\geq 0,$$

$$''='' \text{ iff } a=b=c=1\ (\text{QED})$$