ROMANIAN MATHEMATICAL MAGAZINE

$$If\ x,y,z>0, x+y+z=3\ and\ \lambda\geq 0\ then:$$

$$\sum_{cyc}\frac{x^4}{\sqrt{x^2+\lambda x}}\geq\frac{3}{\sqrt{\lambda+1}}$$

Proposed by Marin Chirciu-Romania

Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum_{cyc}\frac{x^4}{\sqrt{x^2+\lambda x}}=\sum_{cyc}\frac{x^{3,5}}{\sqrt{x+\lambda}}\overset{HOLDER}{\geq}\frac{(\sum x)^{3,5}}{3^{1,5}\sum\sqrt{x+\lambda}}\overset{CBS}{\geq}$$

$$\geq\frac{(\sum x)^{3,5}}{3^{1,5}\sqrt{3\sum(x+\lambda)}}=\frac{3^{3,5}}{3^{1,5}\sqrt{3(x+y+z+3\lambda)}}=$$

$$=\frac{9}{\sqrt{3(3+3\lambda)}}=\frac{3}{\sqrt{\lambda+1}}$$

$$Equality\ holds\ for\ x=y=z=1.$$