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If $a, b, c > 0, abc = 1$ and $\lambda \geq 0$ then :

$$\sum_{\text{cyc}} \frac{1}{a + \lambda ab} \geq \frac{3}{\lambda + 1}$$

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Since $abc = 1$, so we can assign : $a = \frac{x}{y}, b = \frac{y}{z}, c = \frac{z}{x}$ and then :

$$\begin{aligned} \sum_{\text{cyc}} \frac{1}{a + \lambda ab} &= \sum_{\text{cyc}} \frac{1}{\frac{x}{y} + \lambda \cdot \frac{x}{z}} = \sum_{\text{cyc}} \frac{(yz)^2}{zx \cdot yz + \lambda \cdot xy \cdot yz} \geq \frac{(\sum_{\text{cyc}} xy)^2}{xyz(\sum_{\text{cyc}} x)(\lambda + 1)} \geq \\ \frac{3xyz(\sum_{\text{cyc}} x)}{xyz(\sum_{\text{cyc}} x)(\lambda + 1)} &= \frac{3}{\lambda + 1} \quad \forall a, b, c > 0 \mid abc = 1, " = " \text{ iff } a = b = c = 1 \text{ (QED)} \end{aligned}$$