

ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z > 0$, $\sum_{\text{cyc}} x = 1$ and $\lambda \geq 1$ then :

$$\sum_{\text{cyc}} \frac{x^2}{\lambda - x^2} \geq \frac{3}{9\lambda - 1}$$

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$$\lambda - x^2 \geq 1 - x^2 = (1+x)(y+z) > 0 \quad (\because x, y, z > 0)$$

$$\therefore \lambda - x^2 > 0 \text{ and analogs } \therefore \sum_{\text{cyc}} \frac{x^2}{\lambda - x^2} \stackrel{\text{Bergstrom}}{\geq} \frac{(\sum_{\text{cyc}} x)^2}{3\lambda - \sum_{\text{cyc}} x^2} \stackrel{?}{\geq} \frac{3}{9\lambda - 1}$$

$$\Leftrightarrow 9\lambda \left(\left(\sum_{\text{cyc}} x \right)^2 - 1 \right) + 3 \sum_{\text{cyc}} x^2 - \left(\sum_{\text{cyc}} x \right)^2 \stackrel{?}{\geq} 0$$

$$\Leftrightarrow 2 \left(\sum_{\text{cyc}} x^2 - \sum_{\text{cyc}} xy \right) \stackrel{?}{\geq} 0 \quad \left(\because \sum_{\text{cyc}} x = 1 \right) \rightarrow \text{true} \therefore \sum_{\text{cyc}} \frac{x^2}{\lambda - x^2} \geq \frac{3}{9\lambda - 1}$$

$$\forall x, y, z > 0 \mid \sum_{\text{cyc}} x = 1 \text{ and } \lambda \geq 1, " = " \text{ iff } x = y = z = \frac{1}{3} \text{ (QED)}$$