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If $a, b, c > 0$, and $\lambda \geq 0$ then:

$$\sum_{cyc} \frac{ab}{a + \lambda b} \geq \frac{1}{\lambda + 1} \sum_{cyc} a$$

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Solution by Shirvan Tahirov-Azerbaijan

$$\begin{aligned} \sum_{cyc} \frac{ab}{a + \lambda b} &= \sum_{cyc} \left(b - \frac{\lambda b^2}{a + \lambda b} \right) = \sum_{cyc} b - \lambda \sum_{cyc} \frac{b^2}{a + \lambda b} \stackrel{\text{Bergstrom}}{\geq} \\ &\leq \sum_{cyc} a - \lambda \frac{(\sum_{cyc} a)^2}{\sum_{cyc} a + \lambda \sum_{cyc} a} = \sum_{cyc} a - \frac{\lambda \sum_{cyc} a}{1 + \lambda} = \frac{1}{\lambda + 1} \sum_{cyc} a \end{aligned}$$

Equality holds for : $a = b = c$.