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If $a, b, c, x, y, z > 0$, $a + b + c = 3$ and $\lambda \geq 0$ then:

$$\sum_{cyc} \sqrt{1 + \lambda a^2} \geq 3\sqrt{\lambda + 1}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\begin{aligned} \sum_{cyc} \sqrt{1 + \lambda a^2} &= \sqrt{1^2 + (\sqrt{\lambda}a)^2} + \sqrt{1^2 + (\sqrt{\lambda}b)^2} + \sqrt{1^2 + (\sqrt{\lambda}c)^2} \stackrel{\text{Minkowski}}{\geq} \\ &\geq \sqrt{(1 + 1 + 1)^2 + (\sqrt{\lambda}a + \sqrt{\lambda}b + \sqrt{\lambda}c)^2} = \sqrt{9 + \lambda(a + b + c)^2} = 3\sqrt{\lambda + 1} \end{aligned}$$

Equality holds for : $a = b = c = 1$