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If $a, b, c, d > 0, \quad abcd = 1$ *and* $n \in N$ *then*:

$$\sum_{cyc} \frac{1 + a^{2n}}{\sqrt{a}} \geq 8$$

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Solution by Shirvan Tahirov-Azerbaijan

$$\therefore \sum_{cyc} \frac{1 + a^{2n}}{\sqrt{a}} \geq 8$$

$$S = \sum_{cyc} \left(\frac{1}{\sqrt{a}} + \frac{a^{2n}}{\sqrt{a}} \right) \geq \sum_{cyc} \frac{1}{\sqrt{a}} + \sum_{cyc} \frac{a^{2n}}{\sqrt{a}} = S_1 + S_2$$

$$S_1 = \sum_{cyc} \frac{1}{\sqrt{a}} \overset{AM-GM}{\geq} 4\sqrt[4]{\frac{1}{\sqrt{abcd}}} = 4$$

$$S_2 = \sum_{cyc} \frac{a^{2n}}{\sqrt{a}} \overset{AM-GM}{\geq} 4\sqrt[4]{\frac{a^{2n}.b^{2n}.c^{2n}.d^{2n}}{\sqrt{abcd}}} \geq 4\sqrt[4]{(abcd)^{2n-\frac{1}{2}}} = 4$$

$$S \geq S_1 + S_2 \geq 4 + 4 \geq 8$$

$$\textit{Equality holds for}: \quad a = b = c = d = 1.$$