

ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$, $\sum_{\text{cyc}} a = 3$ and $n \in \mathbb{N}$ then :

$$\sum_{\text{cyc}} \frac{a^3}{b^n c} + \sum_{\text{cyc}} a^n \geq 6$$

Proposed by Marin Chirciu-Romania

Solution by Soumava Chakraborty-Kolkata-India

$$\begin{aligned} \sum_{\text{cyc}} \frac{a^3}{b^n c} + \sum_{\text{cyc}} a^n &= \sum_{\text{cyc}} \left(\frac{a^3}{b^n c} + b^n \right) \stackrel{\text{AM-GM}}{\geq} 2 \cdot \sum_{\text{cyc}} \sqrt{\frac{a^3}{b^n c} \cdot b^n} = 2 \cdot \sum_{\text{cyc}} \frac{a \cdot \sqrt{a} \cdot \sqrt{a}}{\sqrt{ca}} \\ &\stackrel{\text{Bergstrom}}{\geq} 2 \cdot \frac{(\sum_{\text{cyc}} a)^2}{\sum_{\text{cyc}} \sqrt{ab}} \stackrel{\text{CBS}}{\geq} 2 \cdot \frac{(\sum_{\text{cyc}} a)^2}{\sqrt{3 \sum_{\text{cyc}} ab}} \geq 2 \cdot \frac{(\sum_{\text{cyc}} a)^2}{\sqrt{(\sum_{\text{cyc}} a)^2}} = 2 \sum_{\text{cyc}} a = 6 \text{ and so,} \\ \sum_{\text{cyc}} \frac{a^3}{b^n c} + \sum_{\text{cyc}} a^n &\geq 6 \quad \forall a, b, c > 0, \sum_{\text{cyc}} a = 3 \text{ and } n \in \mathbb{N}, \\ &'' = '' \text{ iff } a = b = c = 1 \text{ (QED)} \end{aligned}$$