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If $a, b, c > 0, a+b+c=1, \lambda - n = 1, \lambda \geq 3$, then :

$$\sum_{cyc} a^2 \leq \lambda.\sqrt{\frac{\sum_{cyc} a^2}{3}} - n.\sqrt{3abc}$$

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$$\lambda.\sqrt{\frac{\sum_{cyc} a^2}{3}} - n.\sqrt{3abc} \geq \frac{\lambda}{3}.\sum_{cyc} a - (\lambda-1).\sqrt{3abc\left(\sum_{cyc} a\right)}$$

$$\left(\because \sum_{cyc} a = 1 \text{ and } \because n = \lambda - 1\right) \geq \frac{\lambda}{3}.\sum_{cyc} a - (\lambda-1).\sqrt{\left(\sum_{cyc} ab\right)^2}$$

$$(\because \lambda - 1 \geq 2 > 0) = \frac{\lambda}{3}.\left(\sum_{cyc} a\right)^2 - (\lambda-1)\left(\sum_{cyc} ab\right)\left(\because \sum_{cyc} a = 1\right)$$

$$= \frac{\lambda}{3}.\left(\sum_{cyc} a^2 - \sum_{cyc} ab\right) + \sum_{cyc} ab \geq \frac{3}{3}.\left(\sum_{cyc} a^2 - \sum_{cyc} ab\right) + \sum_{cyc} ab$$

$$\left(\because \lambda \geq 3 \text{ and } \because \sum_{cyc} a^2 - \sum_{cyc} ab \geq 0\right) = \sum_{cyc} a^2 \text{ and so,}$$

$$\sum_{cyc} a^2 \leq \lambda.\sqrt{\frac{\sum_{cyc} a^2}{3}} - n.\sqrt{3abc}\ \forall\, a, b, c > 0 \,\middle|\, a+b+c=1, \lambda - n = 1, \lambda \geq 3,$$

$$'' = ''\text{ iff } a = b = c = \frac{1}{3}\ (\text{QED})$$