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If $a,b,c,x,y,z>0,\quad \sum a=\sum x$ and $\lambda\ge 0$ then:

$$\sum_{cyc}\frac{a^2}{\sqrt{\lambda x^2+xy}}\ge\frac{1}{\sqrt{\lambda+1}}\sum x$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum_{cyc}\frac{a^2}{\sqrt{\lambda x^2+xy}}\overset{Bergstrom}{\geq}\frac{(\sum a)^2}{\sum\sqrt{\lambda x^2+xy}}\ge\frac{(\sum x)^2}{\sqrt{3\sum(\lambda x^2+xy)}}\ge\frac{(\sum x)^2}{\sqrt{3(\sum\lambda x^2+\sum xy)}}=$$

$$=\frac{(\sum x)^2}{\sqrt{3(\lambda\sum xy+\sum xy)}}=\frac{1}{\sqrt{\lambda+1}}.\frac{(\sum x)^2}{\sqrt{3\sum xy}}=\frac{1}{\sqrt{\lambda+1}}\sum x$$

Equality holds for: $a=b=c=x=y=z.$