

ROMANIAN MATHEMATICAL MAGAZINE

If $x_1, x_2, x_3 \dots x_n > 0$, $n \geq 3$ then :

$$\sum_{cyc} \frac{x_1 + x_2}{x_3 + x_4 + \dots + x_n} \geq \frac{2n}{n-2}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$S = x_1 + x_2 + x_3 \dots + x_n$$

$$\sum_{cyc} \left(\frac{x_1 + x_2}{x_3 + x_4 + \dots + x_n} + 1 \right) - n = \sum_{cyc} \frac{S}{S - (x_1 + x_2)} - n =$$

$$S \cdot \left(\sum_{cyc} \frac{1^2}{S - (x_1 + x_2)} \right) - n \stackrel{\text{Bergstrom}}{\geq} S \cdot \frac{(1 + 1 + 1 \dots + 1)^2}{\sum_{cyc} S - \sum_{cyc} (x_1 + x_2)} - n =$$

$$= S \cdot \frac{n^2}{nS - 2S} - n = \frac{n^2}{n-2} - n = \frac{2n}{n-2}$$

Equality holds for : $x_1 = x_2 = x_3 \dots = x_n$