If $x, y, z > 0, x + y + z = 3$ *and* $\lambda \geq 0$ then :

$$\sum_{cyc} \frac{x}{y(x + \lambda z)} \geq \frac{3}{\lambda + 1}$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\sum_{cyc} \frac{x}{y(x + \lambda z)} = \sum_{cyc} \frac{\left(\sqrt{\frac{x}{y}}\right)^2}{x + \lambda z} \overset{\text{Bergstrom}}{\geq} \frac{\left(\sum_{cyc} \sqrt{\frac{x}{y}}\right)^2}{\sum_{cyc} x + \lambda \sum_{cyc} x} \overset{\text{AM-GM}}{\geq}$$

$$\frac{\left(3 . \sqrt[3]{\sqrt{\frac{x}{y}} . \sqrt{\frac{y}{z}} . \sqrt{\frac{z}{x}}}\right)^2}{(\lambda + 1)\left(\sum_{cyc} x\right)} \overset{x+y+z=3}{=} \frac{9}{3(\lambda + 1)} \text{ and so, } \sum_{cyc} \frac{x}{y(x + \lambda z)} \geq \frac{3}{\lambda + 1}$$

$\forall\, x, y, z > 0 \,\middle|\, x + y + z = 3$ *and* $\lambda \geq 0$, $'' = ''$ iff $x = y = z = 1$ (QED)