ROMANIAN MATHEMATICAL MAGAZINE

If $a, \mathrm{b}, \mathrm{c} > 0, \sum_{cyc} a^2 = 3a\mathrm{bc}$ and $\mathrm{n} \in \mathbb{N}^*$ then :

$$\sum_{cyc} \frac{a^{\mathrm{n}}}{\mathrm{b}^{\mathrm{n}+1}\mathrm{c}^{\mathrm{n}+1}} \geq \frac{9}{a+\mathrm{b}+\mathrm{c}}$$

Proposed by Marin Chirciu-Romania

Solution by Soumava Chakraborty-Kolkata-India

$$\sum_{cyc} \frac{a^{\mathrm{n}}}{\mathrm{b}^{\mathrm{n}+1}\mathrm{c}^{\mathrm{n}+1}} = \sum_{cyc} \frac{\left(\frac{a}{\mathrm{bc}}\right)^{\mathrm{n}+1}}{a} \overset{Holder}{\geq} \frac{\left(\sum_{cyc} \frac{a}{\mathrm{bc}}\right)^{\mathrm{n}+1}}{3^{\mathrm{n}-1}.\left(\sum_{cyc} a\right)} = \frac{3^{\mathrm{n}+1}}{3^{\mathrm{n}-1}.\left(\sum_{cyc} a\right)}$$

$$\left(\because \sum_{cyc} a^2 = 3a\mathrm{bc} \Rightarrow \sum_{cyc} \frac{a}{\mathrm{bc}} = 3\right) = \frac{9}{a+\mathrm{b}+\mathrm{c}} \text{ and so, } \sum_{cyc} \frac{a^{\mathrm{n}}}{\mathrm{b}^{\mathrm{n}+1}\mathrm{c}^{\mathrm{n}+1}} \geq \frac{9}{a+\mathrm{b}+\mathrm{c}}$$

$$\forall\, a, \mathrm{b}, \mathrm{c} > 0 \,\Big|\, \sum_{cyc} a^2 = 3a\mathrm{bc}, ''='' \text{ iff } a = \mathrm{b} = \mathrm{c} = 1 \text{ (QED)}$$