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Let a, b, c be positive real numbers with $a + b + c \geq 3$.

Prove that:

$$\sum_{cyc} \frac{a^8}{a^2 + nbc} + \frac{\sum_{cyc} a^2}{n+1} \geq \frac{6}{n+1}, \quad n > 0$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\begin{aligned} \sum_{cyc} \frac{a^8}{a^2 + nbc} + \frac{\sum_{cyc} a^2}{n+1} &= \sum_{cyc} \frac{(a^2)^4}{a^2 + nbc} + \frac{\sum_{cyc} a^2}{n+1} \stackrel{\text{Bergstrom}}{\geq} \\ &\geq \frac{(\sum_{cyc} a^2)^4}{9(\sum_{cyc} a^2 + n \sum_{cyc} bc)} + \frac{\sum_{cyc} a^2}{n+1} \geq \frac{(\sum_{cyc} a^2)^4}{9(\sum_{cyc} bc + n \sum_{cyc} bc)} + \frac{\sum_{cyc} a^2}{n+1} = \\ &= \frac{(\sum_{cyc} a^2)^3}{9(n+1)} + \frac{\sum_{cyc} a^2}{n+1} \geq \frac{\left(\frac{(\sum_{cyc} a)^2}{3}\right)^3}{9(n+1)} + \frac{(\sum_{cyc} a)^2}{3(n+1)} = \\ &= \frac{(\sum_{cyc} a)^6}{27 \cdot 9(n+1)} + \frac{(\sum_{cyc} a)^2}{3(n+1)} \geq \frac{3^6}{27 \cdot 9(n+1)} + \frac{9}{3(n+1)} = \frac{3}{n+1} + \frac{3}{n+1} = \frac{6}{n+1} \end{aligned}$$

Equality holds if: $a = b = c = 1$

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