

ROMANIAN MATHEMATICAL MAGAZINE

Let a, b, c be positive real numbers with $a + b + c = 1$

Prove that:

$$\frac{a}{\sqrt{b+c}} + \frac{b}{\sqrt{a+c}} + \frac{c}{\sqrt{b+a}} \geq \sqrt{\frac{3}{2}}$$

Proposed by Kostantinos Geronikolas-Greece

Solution by Mirsadix Muzefferov-Azerbaijan

$$\begin{aligned} \frac{a}{\sqrt{b+c}} + \frac{b}{\sqrt{a+c}} + \frac{c}{\sqrt{b+a}} &= \frac{a^{\frac{3}{2}}}{\sqrt{ab+ac}} + \frac{b^{\frac{3}{2}}}{\sqrt{cb+ab}} + \frac{c^{\frac{3}{2}}}{\sqrt{bc+ac}} \stackrel{\text{Radon}}{\geq} \\ &\geq \frac{(\sum_{\text{cyc}} a)^{\frac{3}{2}}}{(2 \sum_{\text{cyc}} ab)^{\frac{1}{2}}} = \frac{1}{(2 \sum_{\text{cyc}} ab)^{\frac{1}{2}}} \geq \frac{\sqrt{3}}{\sqrt{2} \sum_{\text{cyc}} a} = \sqrt{\frac{3}{2}} \end{aligned}$$

Equality holds if: $a = b = c = \frac{1}{3}$