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$$If\ \ a,b,c>0\ \ and\ \ a+b+c\geq 3\ \ then:$$

$$\frac{a^8}{a^2+nbc}+\frac{b^8}{b^2+nac}+\frac{c^8}{c^2+nab}+\frac{a^2+b^2+c^2}{n+1}\geq\frac{6}{n+1}$$

Proposed by Konstantinos Geronikolas-Greece

Solution by Mirsadix Muzefferov-Azerbaijan

$$(LHS)_1=\sum_{cyc}\frac{a^8}{a^2+nbc}=\sum_{cyc}\frac{(a^2)^4}{a^2+nbc}\overset{CBS}{\geq}\frac{(\sum a^2)^4}{3^2.(\sum a^2+n\sum bc)}\overset{\sum bc\leq\sum a^2}{\geq}$$

$$\geq\frac{(\sum a^2)^4}{9(n+1)\sum a^2}=\frac{(\sum a^2)^3}{9(n+1)}$$

$$(LHS)_1\geq\frac{(\sum a^2)^3}{9(n+1)}\ \ ;\ \ (LHS)_2\geq\frac{\sum a^2}{(n+1)}$$

$$LHS=(LHS)_1+(LHS)_2\geq\frac{(\sum a^2)^3}{9(n+1)}+\frac{\sum a^2}{(n+1)}\overset{AM-GM}{\geq}$$

$$2\sqrt{\frac{(\sum a^2)^3}{9(n+1)}.\frac{\sum a^2}{(n+1)}}=\frac{2}{3(n+1)}\left(\sum a^2\right)^2\geq\frac{2}{3(n+1)}.9=\frac{6}{n+1}$$

$$\therefore\sum_{cyc}a^2\geq\frac{\left(\sum_{cyc}a\right)^2}{3}\geq\frac{9}{3}=3\ \rightarrow\ \sum_{cyc}a^2\geq 3$$

$$Equality\ \ holds\ \ for:\ \ a=b=c=1$$