

ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$ and $a + b + c = 3$ Prove that :

$$\sum_{cyc} \frac{a^3}{a+b+1} \geq 1$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\begin{aligned} \sum_{cyc} \frac{a^3}{a+b+1} &\stackrel{CBS}{\geq} \frac{(a+b+c)^3}{3(a+b+1+b+c+1+c+a+1)} = \\ &= \frac{3^3}{3(2(a+b+c)+3)} = \frac{27}{3(2 \cdot 3 + 3)} = 1 \end{aligned}$$

Equality holds if only if : $a = b = c = 1$