

ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c, d, e > 0$ then:

$$\frac{a+b}{c+d+e} + \frac{b+c}{d+e+a} + \frac{d+c}{b+e+a} + \frac{d+e}{a+b+c} + \frac{e+a}{d+b+c} \geq \frac{10}{3}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\begin{aligned} & \frac{a+b}{c+d+e} + \frac{b+c}{d+e+a} + \frac{d+c}{b+e+a} + \frac{d+e}{a+b+c} + \frac{e+a}{d+b+c} = \\ & = \left(1 + \frac{a+b}{c+d+e}\right) + \left(1 + \frac{b+c}{d+e+a}\right) + \left(1 + \frac{d+c}{b+e+a}\right) + \\ & \quad + \left(1 + \frac{d+e}{a+b+c}\right) + \left(1 + \frac{e+a}{d+b+c}\right) - 5 = \\ & = (a+b+c+d+e) \left(\frac{1}{c+d+e} + \frac{1}{d+e+a} + \frac{1}{b+e+a} + \frac{1}{a+b+c} + \frac{1}{d+b+c} \right) - 5 \stackrel{\text{Bergstrom}}{\geq} \\ & \geq (a+b+c+d+e) \cdot \frac{(1+1+1+1+1)^2}{3(a+b+c+d+e)} - 5 = \frac{25}{3} - 5 = \frac{10}{3} \end{aligned}$$

Equality holds for : $a = b = c = d = e$.