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PROBLEMS FOR JUNIORS

JP.601. If $a, b, c > 0$ and $\lambda \geq 0$ then:

$$(\lambda + 1)\sum \frac{a^2}{b + \lambda c} + 2\sqrt{3(ab + bc + ca)} \geq 3(a + b + c)$$

Proposed by Marin Chirciu - Romania

JP.602. If $a, b, c > 0$ and $\lambda \geq 0$ then:

$$2\sum \frac{a^3}{b + \lambda c} + \frac{\lambda + 1}{2}\sum bc \geq 2\sum a^2$$

Proposed by Marin Chirciu - Romania

JP.603. If $a, b, c > 0$ then:

$$2(a^a + b^b + c^c) \geq a^b + a^c + b^a + b^c + c^a + c^b$$

Proposed by Daniel Sitaru - Romania

JP.604. If $a, b > 0; n \in \mathbb{N}^*$ then:

$$\frac{a^{2n}}{b^{2n}} + a^{2n}b^{4n} + \frac{1}{a^{4n}b^{2n}} \geq \frac{b}{a} + a^2b + \frac{1}{ab^2}$$

Proposed by Daniel Sitaru - Romania

JP.605. Find if it exists $x, y \in \mathbb{R}^*$ such that:

$$4^{x+y} + \sqrt{2026^x + 2026^y - 2} = 2^{x+y+1} - 1$$

Proposed by Adrian Gobej - Romania

JP.606. Solve the following equation:

$$2016^{2-x^2} + 2016^{2017x^2 - 4\sqrt{x} - 2013} = 2017$$

Proposed by Adrian Gobej - Romania

JP.607. Solve for reals:

$$\sqrt[4]{8x - 7} + \sqrt[4]{3 - 2x^4} = 2$$

Proposed by Marin Chirciu - Romania

JP.608. **Let be the triangle ABC, AA_1, BB_1, CC_1 internal bisectors and A_2, B_2, C_2 contact points to the bisectors with the circumcircle of the triangle. Prove that:**

$$A_1A_2 \cdot B_2C_2 + B_1B_2 \cdot A_2C_2 + C_1C_2 \cdot A_2B_2 \geq Rs$$

where p represent the semiperimeter and R the circumradii of triangle ABC.

Proposed by Marian Ursărescu, Florică Anastase - Romania

JP.609. **In acute triangle ABC, A', B', C' are symmetric points of the points A, B, C to the sides BC, AC, and AB respectively. Prove that:**

$$\frac{\rho[A'B'C']}{\rho[ABC]} = 4\left(\frac{r}{R}\right)^2 + 8 \cdot \frac{r}{R} - 1$$

where $\rho[ABC]$ represent area of ΔABC.

Proposed by Marian Ursărescu, Florică Anastase - Romania

JP.610. **Solve for real numbers:**

$$\log_{2\sqrt{8+2\sqrt{15}}}(x^2 + x + 2) = \log_{\sqrt{4+\sqrt{15}}}(x^2 + x + 1)$$

Proposed by Marian Ursărescu, Florică Anastase - Romania

JP.611. **Find the angle between the real plans:**

$$P_1 : 2x + y + 3z - 1 = 0$$

$$P_2 : 3x - 2y + z + 1 = 0$$

Proposed by Daniel Sitaru - Romania

JP.612. **Find the angle between the line d and the real plan P.**

$$d : \begin{cases} x = 2 + 3t \\ y = 3 - 2t; \\ z = 1 + 5t \end{cases} ; t \in \mathbb{R}; P : 2x + y + z - 5 = 0$$

Proposed by Daniel Sitaru - Romania

JP.613. **Find the angle between the lines:**

$$d_1 : \begin{cases} x = 3 - 2t \\ y = 1 + t \\ z = -1 + 3t \end{cases} ; d_2 : \begin{cases} x = 2 + 4t \\ y = -1 + 2t; \\ z = 4 - t \end{cases} ; t \in \mathbb{R}$$

Proposed by Daniel Sitaru - Romania

JP.614. In ΔABC the following relationship holds:

$$\frac{a^4+b^4}{h_c}+\frac{b^4+c^4}{h_a}+\frac{c^4+a^4}{h_b}\geq 288r^3$$

Proposed by Nguyen Hung Cuong - Romania

JP.615. In ΔABC the following relationship holds:

$$\frac{a^4+b^4}{h_c^2}+\frac{b^4+c^4}{h_a^2}+\frac{c^4+a^4}{h_b^2}\geq 96r^2$$

Proposed by Nguyen Hung Cuong - Romania

PROBLEMS FOR SENIORS

SP.601. What is the largest positive value of the power k such that

$$a^k+b^k+c^k\leq 3$$

for all nonnegative real numbers a,b,c,d with $a\leq b\leq c\leq d$ and

$$ab+bc+cd+da=4?$$

Proposed by Vasile Cîrtoaje - Romania

SP.602. Let $a_1,a_2,\ldots,a_n$ be nonnegatice real numbers such that $a_1+a_2+\ldots+a_n=n$. Prove that:

$$\sum_{i=1}^{n}\sqrt{\frac{n-a_i}{n-1+a_i}}\geq\sqrt{n(n-1)}$$

Proposed by Vasile Cîrtoaje - Romania

SP.603. Prove that $\frac{5}{3}$ is the largest positive value of the power k such that

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\geq a^k+b^k+c^k+d^k$$

for all positive real numbers a,b,c,d with at most one of them smaller than 1 and

$$ab+ac+ad+bc+bd+cd=6$$

Proposed by Vasile Cîrtoaje - Romania

SP.604. Let be $A(2,1,0);B(1,2,1);C(3,3,3);D(0,0,4)$. Find the distance from the point D to the real plan (ABC).

Proposed by Daniel Sitaru - Romania

SP.605. Let be $A(1,2,1);B(2,1,3);C(4,4,4)$. Find the distance from the point A to the line BC.

Proposed by Daniel Sitaru - Romania

SP.606. Let be $A(2,1,0); B(0,1,2); C(3,0,1); D(4,4,4)$ Find the volume of the tetrahedron $ABCD$.

Proposed by Daniel Sitaru - Romania

SP.607. Let be the function $f:[0,1]\to\mathbb{R}$ integrable such that $f(1)=1$ and

$$\int_x^y f(t)dt=\frac{1}{2}(yf(y)-xf(x)), \forall x,y\in[0,1]$$

Find:

$$I=\int_0^{\frac{\pi}{4}} f(x)\cdot\tan^2 x dx$$

Proposed by Marian Ursărescu, Florică Anastase - Romania

SP.608. If $a,b,c\in(0,1)$ and $x,y,z>0$ such that $a=(bc)^x$, $b=(ca)^y, c=(ab)^z$ and $xyz=1$ then holds:

$$\sqrt[n]{\sum_{cyc} a^n(y+z+2)^{2n-1}}\geq 6\cdot\sqrt[3]{abc}, n\in\mathbb{N}^*, n\geq 2$$

Proposed by Marian Ursărescu, Florică Anastase - Romania

SP.609. If $a,b,c>0$ then:

$$\sqrt{a^2b^2+c^2}+\sqrt{c^2b^2+a^2}+\sqrt{c^2a^2+b^2}\geq\sqrt{(a+b+c)^2+(ab+bc+ca)^2}$$

Proposed by Daniel Sitaru - Romania

SP.610. If $a,b,c>0$ then:

$$\sqrt{a^2+1}+\sqrt{b^2+1}+\sqrt{c^2+1}\geq\sqrt{(a+b+c)^2+9}$$

Proposed by Daniel Sitaru - Romania

SP.611. If $x,y,z>0$ then:

$$(x^5+y^5+z^5)(x^6+y^6+z^6)(x^2+y^2+z^2)^5\geq(x^3+y^3+z^3)^7$$

Proposed by Daniel Sitaru - Romania

SP.612. Let be $A(1,2,3); B(4,1,1); C(0,3,2); D(2,1,3)$. Find the volume of the parallelepiped builded on the vectors $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$.

Proposed by Daniel Sitaru - Romania

SP.613. Let be $A(2,4,1); B(4,2,5)$. Find the mediator plan of the segment $[AB]$.

Proposed by Daniel Sitaru - Romania

SP.614. Let be $A(1,3,2); B(3,1,1); C(4,2,0)$. Find the area of the parallelogram builded on $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

Proposed by Daniel Sitaru - Romania

SP.615. Let be $A(1,2,3)$ and the real plan: $P: 2x+y+z-5=0$. Find the parametrical equations of the perpendicular line from A to the real plan P.

Proposed by Daniel Sitaru - Romania

UNDERGRADUATE PROBLEMS

UP.601. Let $\{L_n\}_{n\geq 0}$ be the Luca's sequence defined by $L_0=2, L_1=1$ and for all $n\geq 2, L_n=L_{n-1}+L_{n-2}$. Determine the sum of the lengths of the intervals, disjoint two by two, formed by all $f(x)=1$, where

$$f(x)=\frac{L_1^2}{x+L_1^2}+\frac{L_2^2}{x+L_2^2}+\ldots+\frac{L_n^2}{x+L_n^2}$$

Proposed by Jose Luiz Diaz Barrero - Spain

UP.602. Find all non-negative integers for which

$$\sum_{k=0}^{n}\frac{k}{k+1}\binom{n}{k}^2$$

is an integer number.

Proposed by Jose Luiz Diaz Barrero - Spain

UP.603. Find all positive real values of the constant k such that

$$9(a^2+k)(b^2+k)(c^2+k)\leq(1+k)^3(a+b+c)^2$$

for any nonnegative real numbers a,b,c with $ab+bc+ca=3$.

Proposed by Vasile Cîrtoaje - Romania

UP.604. Prove that $\frac{3}{2}$ is the smallest positive value of the power k such that

$$\frac{1}{a^k}+\frac{1}{b^k}+\frac{1}{c^k}\geq a^2+b^2+c^2$$

for all positive real numbers a,b,c with at most one of them smaller than 1 and

$$ab+bc+ca=3$$

Proposed by Vasile Cîrtaje - Romania

UP.605. We consider the function $u : \mathbb{R} \to \mathbb{R}$, periodic with period 2π. For the period $[0, 2\pi]$ we have:

$u(x) = \cos(x)$ if $x \in \left[0, \frac{\pi}{2}\right)$; $u(x) = 0$ if $x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right)$; $u(x) = \cos(x)$ if $x \in \left[\frac{3\pi}{2}, 2\pi\right)$

Prove the equality:

$$\int_0^\infty \frac{u(x)}{1+x^2}dx = \frac{\pi}{4e} + \frac{e^2+1}{2e}\arctan\left(\frac{1}{e}\right)$$

Proposed by Vasile Mircea Popa - Romania

UP.606. If $f : \mathbb{R} \to (1, \infty)$ and $g : \mathbb{R} \to \mathbb{R}$ are continuous functions, and $y_n = \sqrt[n]{n!F_n}, n \in \mathbb{N}^* - \{1\}$, where $(F_n)_{n \geq 0}$ is Fibonacci sequence, find:

$$\lim_{n\to\infty} \int_{y_n}^{y_n+1} \frac{(f(x-y_n))^{g(y_{n+1}-x)}}{(f(y_{n+1}-x))^{g(x-y_n)} + (f(x-y_n))^{g(y_{n+1}-x)}}$$

Proposed by D.M. Bătineţu - Giurgiu, Neculai Stanciu - Romania

UP.607. Calculate the integral:

$$\int_0^1 \frac{x \ln x}{(x+1)(x^2+1)}dx$$

Proposed by Vasile Mircea Popa - Romania

UP.608. If $f : \mathbb{R} \to (1, \infty)$ and $f : \mathbb{R} \to \mathbb{R}$ are continuous functions, and $(L_n)_{n \geq 0}$ is Lucas sequence, and
$y_n = \sqrt[n]{(2n-1)!!L_n}, n \in \mathbb{N}^* - \{1\}$, find:

$$\lim_{n\to\infty} \int_{y_n}^{y_{n+1}} \frac{(f(x-y_n))^{g(y_{n+1}-x)}}{(f(y_{n+1}-x))^{g(x-y_n)}(f(x-y_n))^{g(y_{n+1}-x)}}dx$$

Proposed by D.M. Bătineţu - Giurgiu, Neculai Stanciu - Romania

UP.609. If $f : [a, b] \to (0, \infty)$ is continuous function such that $f(a+b-x) + f(x) = c, \forall x \in [a, b]$ and $a + b = \frac{\pi}{2}$, find:

$$\int_a^b \frac{\sin^n x + f(x) + d}{\sin^n x + \cos^n x + c + 2d}dx,$$

where $n \in \mathbb{N}^*$ and $d \geq 0$.

Proposed by D.M. Bătineţu - Giurgiu, Neculai Stanciu - Romania

UP.610. If $f:\mathbb{R}_+^*\to\mathbb{R}_+^*$ is continuous function and $\gamma_n=-\ln n+\sum_{k=1}^n\frac{1}{k}$, find

$$\lim_{n\to\infty} n\int_\gamma^{\gamma_n}\frac{f(x-y\gamma}{f(\gamma_n-x)+f(x-\gamma)}dx$$

Proposed by D.M. Bătineţu - Giurgiu, Neculai Stanciu - Romania

UP.611. If $f:\mathbb{R}_+^*\to\mathbb{R}_+^*$ is continuous function and $(x_n)_{n\geq1}$, $x_n=\sum_{k=1}^n\frac{1}{k}$, find:

$$\lim_{n\to\infty}\int_{e^{x_n}}^{e^{x_n+1}}\frac{f(x-e^{x_n})}{f(e^{x_n+1}-x)+f(x-e^{x_n})}dx$$

Proposed by D.M. Bătineţu - Giurgiu, Neculai Stanciu - Romania

UP.612. If $f:\mathbb{R}_+^*\to\mathbb{R}_+^*$ is a continuous function and $(a_n)_{n\geq1}$ is defined by $a_1=a_2=1, a_{n+1}=\sum_{k=1}^n\frac{a_k}{k}, \forall n\geq 2$ and $(x_n)_{n\geq1}$, $x_n=\sum_{k=1}^n\frac{1}{a_k}$, find

$$\lim_{n\to\infty}\frac{1}{n}\int_{e^{x_n}}^{e^{x_{n+1}}}\frac{f(x-e^{x_n})}{f(e^{x_{n+1}}-x)+f(x-e^{x_n})}dx$$

Proposed by D.M. Bătineţu - Giurgiu, Neculai Stanciu - Romania

UP.613. Let be $A(1,2,0);B(2,0,1);C(3,3,3)$. Find the area of ΔABC.

Proposed by Daniel Sitaru - Romania

UP.614. If $0<a\leq b$ then:

$$\int_a^b\int_a^b\frac{dxdy}{x^2-xy+y^2}+3\Big(\ln\Big(\frac{b}{a}\Big)\Big)^2\geq 16\int_a^b\int_a^b\frac{dxdy}{(x+y)^2}$$

Proposed by Daniel Sitaru - Romania

UP.615. Prove that $\frac{8}{5}$ is the smallest positive value of the constant k such that

$$\frac{1}{(a+b)^2+k}+\frac{1}{(b+c)^2+k}+\frac{1}{(c+a)^2+k}\leq\frac{3}{4+k}$$

for all side lengths a,b,c of a triangle with $ab+bc+ca=3$

Proposed by Vasile Cîrtoaje - Romania

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