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# PROBLEMS FOR JUNIORS

**JP.601.** If $a, b, c > 0$ and $\lambda \geq 0$ then:

$$(\lambda+1)\sum \frac{a^2}{b+\lambda c} + 2\sqrt{3(ab+bc+ca)} \geq 3(a+b+c)$$

*Proposed by Marin Chirciu - Romania*

**JP.602.** If $a, b, c > 0$ and $\lambda \geq 0$ then:

$$2\sum \frac{a^3}{b+\lambda c} + \frac{\lambda+1}{2}\sum bc \geq 2\sum a^2$$

*Proposed by Marin Chirciu - Romania*

**JP.603.** If $a, b, c > 0$ then:

$$2(a^a + b^b + c^c) \geq a^b + a^c + b^a + b^c + c^a + c^b$$

*Proposed by Daniel Sitaru - Romania*

**JP.604.** If $a, b > 0; n \in \mathbb{N}^*$ then:

$$\frac{a^{2n}}{b^{2n}} + a^{2n}b^{4n} + \frac{1}{a^{4n}b^{2n}} \geq \frac{b}{a} + a^2b + \frac{1}{ab^2}$$

*Proposed by Daniel Sitaru - Romania*

**JP.605.** Find if it exists $x, y \in \mathbb{R}^*$ such that:

$$4^{x+y} + \sqrt{2026^x + 2026^y - 2} = 2^{x+y+1} - 1$$

*Proposed by Adrian Gobej - Romania*

**JP.606.** Solve the following equation:

$$2016^{2-x^2} + 2016^{2017x^2 - 4\sqrt{x} - 2013} = 2017$$

*Proposed by Adrian Gobej - Romania*

**JP.607.** Solve for reals:

$$\sqrt[4]{8x-7} + \sqrt[4]{3-2x^4} = 2$$

*Proposed by Marin Chirciu - Romania*

**JP.608.** **Let be the triangle $ABC, AA_1, BB_1, CC_1$ internal bisectors and $A_2, B_2, C_2$ contact points to the bisectors with the circumcircle of the triangle. Prove that:**

$$A_1A_2 \cdot B_2C_2 + B_1B_2 \cdot A_2C_2 + C_1C_2 \cdot A_2B_2 \geq Rs$$

**where $p$ represent the semiperimeter and $R$ the circumradii of triangle $ABC$.**

*Proposed by Marian Ursărescu, Florică Anastase - Romania*

**JP.609.** **In acute triangle $ABC, A', B', C'$ are symmetric points of the points $A, B, C$ to the sides $BC, AC$, and $AB$ respectively. Prove that:**

$$\frac{\rho[A'B'C']}{\rho[ABC]} = 4\left(\frac{r}{R}\right)^2 + 8 \cdot \frac{r}{R} - 1$$

**where $\rho[ABC]$ represent area of $\Delta ABC$.**

*Proposed by Marian Ursărescu, Florică Anastase - Romania*

**JP.610.** **Solve for real numbers:**

$$\log_{2\sqrt{8+2\sqrt{15}}}(x^2 + x + 2) = \log_{\sqrt{4+\sqrt{15}}}(x^2 + x + 1)$$

*Proposed by Marian Ursărescu, Florică Anastase - Romania*

**JP.611.** **Find the angle between the real plans:**

$$P_1 : 2x + y + 3z - 1 = 0$$

$$P_2 : 3x - 2y + z + 1 = 0$$

*Proposed by Daniel Sitaru - Romania*

**JP.612.** **Find the angle between the line $d$ and the real plan $P$.**

$$d : \begin{cases} x = 2 + 3t \\ y = 3 - 2t; \\ z = 1 + 5t \end{cases} ; t \in \mathbb{R}; P : 2x + y + z - 5 = 0$$

*Proposed by Daniel Sitaru - Romania*

**JP.613.** **Find the angle between the lines:**

$$d_1 : \begin{cases} x = 3 - 2t \\ y = 1 + t \\ z = -1 + 3t \end{cases} ; d_2 : \begin{cases} x = 2 + 4t \\ y = -1 + 2t; \\ z = 4 - t \end{cases} ; t \in \mathbb{R}$$

*Proposed by Daniel Sitaru - Romania*


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**JP.614.** In $\Delta ABC$ the following relationship holds:

$$\frac{a^4+b^4}{h_c}+\frac{b^4+c^4}{h_a}+\frac{c^4+a^4}{h_b}\geq 288r^3$$

*Proposed by Nguyen Hung Cuong - Romania*

**JP.615.** In $\Delta ABC$ the following relationship holds:

$$\frac{a^4+b^4}{h_c^2}+\frac{b^4+c^4}{h_a^2}+\frac{c^4+a^4}{h_b^2}\geq 96r^2$$

*Proposed by Nguyen Hung Cuong - Romania*

# PROBLEMS FOR SENIORS

**SP.601.** What is the largest positive value of the power $k$ such that

$$a^k+b^k+c^k\leq 3$$

for all nonnegative real numbers $a,b,c,d$ with $a\leq b\leq c\leq d$ and

$$ab+bc+cd+da=4?$$

*Proposed by Vasile Cîrtoaje - Romania*

**SP.602.** Let $a_1,a_2,\ldots,a_n$ be nonnegatice real numbers such that $a_1+a_2+\ldots+a_n=n$. Prove that:

$$\sum_{i=1}^{n}\sqrt{\frac{n-a_i}{n-1+a_i}}\geq\sqrt{n(n-1)}$$

*Proposed by Vasile Cîrtoaje - Romania*

**SP.603.** Prove that $\frac{5}{3}$ is the largest positive value of the power $k$ such that

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\geq a^k+b^k+c^k+d^k$$

for all positive real numbers $a,b,c,d$ with at most one of them smaller than 1 and

$$ab+ac+ad+bc+bd+cd=6$$

*Proposed by Vasile Cîrtoaje - Romania*

**SP.604.** Let be $A(2,1,0);B(1,2,1);C(3,3,3);D(0,0,4)$. Find the distance from the point $D$ to the real plan $(ABC)$.

*Proposed by Daniel Sitaru - Romania*

**SP.605.** Let be $A(1,2,1);B(2,1,3);C(4,4,4)$. Find the distance from the point $A$ to the line $BC$.

*Proposed by Daniel Sitaru - Romania*

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**SP.606.** **Let be $A(2,1,0); B(0,1,2); C(3,0,1); D(4,4,4)$ Find the volume of the tetrahedron $ABCD$.**

*Proposed by Daniel Sitaru - Romania*

**SP.607.** **Let be the function $f:[0,1]\to\mathbb{R}$ integrable such that $f(1)=1$ and**

$$\int_x^y f(t)dt=\frac{1}{2}(yf(y)-xf(x)),\forall x,y\in[0,1]$$

**Find:**

$$I=\int_0^{\frac{\pi}{4}} f(x)\cdot\tan^2 x dx$$

*Proposed by Marian Ursărescu, Florică Anastase - Romania*

**SP.608.** **If $a,b,c\in(0,1)$ and $x,y,z>0$ such that $a=(bc)^x$, $b=(ca)^y, c=(ab)^z$ and $xyz=1$ then holds:**

$$\sqrt[n]{\sum_{cyc} a^n(y+z+2)^{2n-1}}\geq 6\cdot\sqrt[3]{abc}, n\in\mathbb{N}^*, n\geq 2$$

*Proposed by Marian Ursărescu, Florică Anastase - Romania*

**SP.609.** **If $a,b,c>0$ then:**

$$\sqrt{a^2b^2+c^2}+\sqrt{c^2b^2+a^2}+\sqrt{c^2a^2+b^2}\geq\sqrt{(a+b+c)^2+(ab+bc+ca)^2}$$

*Proposed by Daniel Sitaru - Romania*

**SP.610.** **If $a,b,c>0$ then:**

$$\sqrt{a^2+1}+\sqrt{b^2+1}+\sqrt{c^2+1}\geq\sqrt{(a+b+c)^2+9}$$

*Proposed by Daniel Sitaru - Romania*

**SP.611.** **If $x,y,z>0$ then:**

$$(x^5+y^5+z^5)(x^6+y^6+z^6)(x^2+y^2+z^2)^5\geq(x^3+y^3+z^3)^7$$

*Proposed by Daniel Sitaru - Romania*

**SP.612.** **Let be $A(1,2,3); B(4,1,1); C(0,3,2); D(2,1,3)$. Find the volume of the parallelepiped builded on the vectors $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$.**

*Proposed by Daniel Sitaru - Romania*

**SP.613.** **Let be $A(2,4,1); B(4,2,5)$. Find the mediator plan of the segment $[AB]$.**

*Proposed by Daniel Sitaru - Romania*


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**SP.614. Let be $A(1,3,2); B(3,1,1); C(4,2,0)$. Find the area of the parallelogram builded on $\overrightarrow{AB}$ and $\overrightarrow{AC}$.**

*Proposed by Daniel Sitaru - Romania*

**SP.615. Let be $A(1,2,3)$ and the real plan: $P: 2x+y+z-5=0$. Find the parametrical equations of the perpendicular line from $A$ to the real plan $P$.**

*Proposed by Daniel Sitaru - Romania*

# UNDERGRADUATE PROBLEMS

**UP.601. Let $\{L_n\}_{n\geq 0}$ be the Lucas' sequence defined by $L_0=2, L_1=1$ and for all $n\geq 2, L_n=L_{n-1}+L_{n-2}$. Determine the sum of the lengths of the intervals, disjoint two by two, formed by all $f(x)=1$, where**

$$f(x)=\frac{L_1^2}{x+L_1^2}+\frac{L_2^2}{x+L_2^2}+\ldots+\frac{L_n^2}{x+L_n^2}$$

*Proposed by Jose Luis Diaz Barrero - Spain*

**UP.602. Find all non-negative integers for which**

$$\sum_{k=0}^{n}\frac{k}{k+1}\binom{n}{k}^2$$

**is an integer number.**

*Proposed by Jose Luis Diaz Barrero - Spain*

**UP.603. Find all positive real values of the constant $k$ such that**

$$9(a^2+k)(b^2+k)(c^2+k)\leq(1+k)^3(a+b+c)^2$$

**for any nonnegative real numbers $a,b,c$ with $ab+bc+ca=3$.**

*Proposed by Vasile Cîrtoaje - Romania*

**UP.604. Prove that $\frac{3}{2}$ is the smallest positive value of the power $k$ such that**

$$\frac{1}{a^k}+\frac{1}{b^k}+\frac{1}{c^k}\geq a^2+b^2+c^2$$

**for all positive real numbers $a,b,c$ with at most one of them smaller than 1 and**

$$ab+bc+ca=3$$

*Proposed by Vasile Cîrtaje - Romania*

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**UP.605. We consider the function $u:\mathbb{R}\to\mathbb{R}$, periodic with period $2\pi$. For the period $[0,2\pi]$ we have:**

$$u(x)=\cos(x) \text{ if } x\in\Big[0,\frac{\pi}{2}\Big); u(x)=0 \text{ if } x\in\Big[\frac{\pi}{2},\frac{3\pi}{2}\Big); u(x)=\cos(x) \text{ if } x\in\Big[\frac{3\pi}{2},2\pi\Big)$$

**Prove the equality:**

$$\int_0^{\infty}\frac{u(x)}{1+x^2}dx=\frac{\pi}{4e}+\frac{e^2+1}{2e}\arctan\Big(\frac{1}{e}\Big)$$

*Proposed by Vasile Mircea Popa - Romania*

**UP.606. If $f:\mathbb{R}\to(1,\infty)$ and $g:\mathbb{R}\to\mathbb{R}$ are continuous functions, and $y_n=\sqrt[n]{n!F_n}, n\in\mathbb{N}^*-\{1\}$, where $(F_n)_{n\geq 0}$ is Fibonacci sequence, find:**

$$\lim_{n\to\infty}\int_{y_n}^{y_n+1}\frac{(f(x-y_n))^{g(y_{n+1}-x)}}{(f(y_{n+1}-x))^{g(x-y_n)}+(f(x-y_n))^{g(y_{n+1}-x)}}$$

*Proposed by D.M. Bătineţu - Giurgiu, Neculai Stanciu - Romania*

**UP.607. Calculate the integral:**

$$\int_0^1\frac{x\ln x}{(x+1)(x^2+1)}dx$$

*Proposed by Vasile Mircea Popa - Romania*

**UP.608. If $f:\mathbb{R}\to(1,\infty)$ and $f:\mathbb{R}\to\mathbb{R}$ are continuous functions, and $(L_n)_{n\geq 0}$ is Lucas sequence, and $y_n=\sqrt[n]{(2n-1)!!L_n}, n\in\mathbb{N}^*-\{1\}$, find:**

$$\lim_{n\to\infty}\int_{y_n}^{y_{n+1}}\frac{(f(x-y_n))^{g(y_{n+1}-x)}}{(f(y_{n+1}-x))^{g(x-y_n)}(f(x-y_n))^{g(y_{n+1}-x)}}dx$$

*Proposed by D.M. Bătineţu - Giurgiu, Neculai Stanciu - Romania*

**UP.609. If $f:[a,b]\to(0,\infty)$ is continuous function such that $f(a+b-x)+f(x)=c, \forall x\in[a,b]$ and $a+b=\frac{\pi}{2}$, find:**

$$\int_a^b\frac{\sin^n x+f(x)+d}{\sin^n x+\cos^n x+c+2d}dx,$$

**where $n\in\mathbb{N}^*$ and $d\geq 0$.**

*Proposed by D.M. Bătineţu - Giurgiu, Neculai Stanciu - Romania*

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**UP.610. If $f:\mathbb{R}_+^*\to\mathbb{R}_+^*$ is continuous function and $\gamma_n=-\ln n+\sum_{k=1}^{n}\frac{1}{k}$, find**

$$\lim_{n\to\infty} n\int_{\gamma}^{\gamma_n}\frac{f(x-y\gamma}{f(\gamma_n-x)+f(x-\gamma)}dx$$

*Proposed by D.M. Bătineţu - Giurgiu, Neculai Stanciu - Romania*

**UP.611. If $f:\mathbb{R}_+^*\to\mathbb{R}_+^*$ is continuous function and $(x_n)_{n\geq 1}$, $x_n=\sum_{k=1}^{n}\frac{1}{k}$, find:**

$$\lim_{n\to\infty}\int_{e^{x_n}}^{e^{x_n+1}}\frac{f(x-e^{x_n})}{f(e^{xn+1}-x)+f(x-e^{x_n})}dx$$

*Proposed by D.M. Bătineţu - Giurgiu, Neculai Stanciu - Romania*

**UP.612. If $f:\mathbb{R}_+^*\to\mathbb{R}_+^*$ is a continuous function and $(a_n)_{n\geq 1}$ is defined by $a_1=a_2=1, a_{n+1}=\sum_{k=1}^{n}\frac{a_k}{k}, \forall n\geq 2$ and $(x_n)_{n\geq 1}$, $x_n=\sum_{k=1}^{n}\frac{1}{a_k}$, find**

$$\lim_{n\to\infty}\frac{1}{n}\int_{e^{x_n}}^{e^{x_{n+1}}}\frac{f(x-e^{x_n})}{f(e^{x_{n+1}}-x)+f(x-e^{x_n})}dx$$

*Proposed by D.M. Bătineţu - Giurgiu, Neculai Stanciu - Romania*

**UP.613. Let be $A(1,2,0); B(2,0,1); C(3,3,3)$. Find the area of $\Delta ABC$.**

*Proposed by Daniel Sitaru - Romania*

**UP.614. If $0<a\leq b$ then:**

$$\int_a^b\int_a^b\frac{dxdy}{x^2-xy+y^2}+3\Big(\ln\Big(\frac{b}{a}\Big)\Big)^2\geq 16\int_a^b\int_a^b\frac{dxdy}{(x+y)^2}$$

*Proposed by Daniel Sitaru - Romania*

**UP.615. Prove that $\frac{8}{5}$ is the smallest positive value of the constant $k$ such that**

$$\frac{1}{(a+b)^2+k}+\frac{1}{(b+c)^2+k}+\frac{1}{(c+a)^2+k}\leq\frac{3}{4+k}$$

**for all side lengths $a,b,c$ of a triangle with $ab+bc+ca=3$**

*Proposed by Vasile Cîrtoaje - Romania*

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