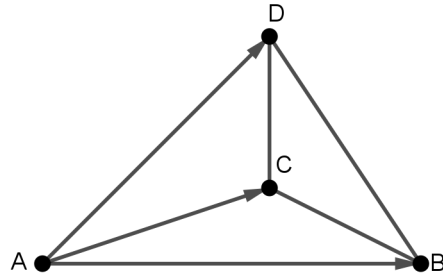


VOLUME OF THE TETRAHEDRON

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Let be $A(1, 1, 1); B(1, 2, 3); C(2, 3, 1); D(4, 4, 4)$.

Find the volume of the tetrahedron $ABCD$.

$$\begin{aligned}\vec{AB} &= (x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} + (z_B - z_A)\vec{k} = \\ &= (1 - 1)\vec{i} + (2 - 1)\vec{j} + (3 - 1)\vec{k} = \vec{j} + 2\vec{k} \\ \vec{AC} &= (x_C - x_A)\vec{i} + (y_C - y_A)\vec{j} + (z_C - z_A)\vec{k} = \\ &= (2 - 1)\vec{i} + (3 - 1)\vec{j} + (1 - 1)\vec{k} = \vec{i} + 2\vec{j} \\ \vec{AB} \times \vec{AC} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{vmatrix} = 2\vec{j} - \vec{k} - 4\vec{i} = -4\vec{i} + 2\vec{j} - \vec{k} \\ \vec{AD} &= (x_D - x_A)\vec{i} + (y_D - y_A)\vec{j} + (z_D - z_A)\vec{k} = \\ &= (4 - 1)\vec{i} + (4 - 1)\vec{j} + (4 - 1)\vec{k} = 3\vec{i} + 3\vec{j} + 3\vec{k} \\ \vec{AD} \cdot (\vec{AB} \times \vec{AC}) &= -4 \cdot 3 + 2 \cdot 3 - 1 \cdot 3 = -12 + 6 - 3 = -9 \\ V[ABCD] &= \frac{1}{6} |\vec{AD} \cdot (\vec{AB} \times \vec{AC})| = \frac{1}{6} |-9| = \frac{3}{2}\end{aligned}$$

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