THE DISTANCE FROM A POINT TO A LINE

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Let be $A(2,-1,1); B(0,1,3); C(-1,2,2)$.
Find the distance from the point A to the line BC.

$$\overrightarrow{AB} = (x_B - x_A)\overrightarrow{i} + (y_b - y_A)\overrightarrow{j} + (z_B - z_A)\overrightarrow{k} =$$
$$= (0-2)\overrightarrow{i} + (1+1)\overrightarrow{j} + (3-1)\overrightarrow{k} = -2\overrightarrow{i} + 2\overrightarrow{j} + 2\overrightarrow{k}$$
$$\overrightarrow{AC} = (x_C - x_A)\overrightarrow{i} + (y_C - y_A)\overrightarrow{j} + (z_C - z_A)\overrightarrow{k}$$
$$= (-1-2)\overrightarrow{i} + (2+1)\overrightarrow{j} + (2-1)\overrightarrow{k} = -3\overrightarrow{i} + 3\overrightarrow{j} + \overrightarrow{k}$$
$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\ -2 & 2 & 2 \\ -3 & 3 & 1 \end{vmatrix} = 2\overrightarrow{i} - 6\overrightarrow{j} - 6\overrightarrow{k} + 6\overrightarrow{k} - 6\overrightarrow{i} + 2\overrightarrow{j}$$
$$\overrightarrow{AB} \times \overrightarrow{AC} = -4\overrightarrow{i} - 4\overrightarrow{j}$$
$$A[ABC] = \frac{1}{2}\sqrt{(-4)^2 + (-4)^2} = \frac{32}{2} = 2\sqrt{2}$$
$$BC = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2 + (z_C - z_B)^2} =$$
$$= \sqrt{(-1-0)^2 + (2-1)^2 + (2-3)^2} = \sqrt{3}$$
$$d(A, BC) = \frac{2A[ABC]}{BC} = \frac{2 \cdot 2\sqrt{2}}{\sqrt{3}} = \frac{4\sqrt{2}}{\sqrt{3}} = \frac{4\sqrt{6}}{3}$$

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