ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{a}{b}\sqrt{a^2+b^2}+\frac{b}{c}\sqrt{b^2+c^2}+\frac{c}{a}\sqrt{c^2+a^2}>\frac{1}{\sqrt{2}}\left(\frac{a^2-b^2}{a+3b}+\frac{b^2-c^2}{b+3c}+\frac{c^2-a^2}{c+3a}\right)$$

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Solution by Jenish Rijal-Nepal

$$\sqrt{a^2+b^2}\overbrace{\geq}^{RMS-AM}\frac{a+b}{\sqrt{2}}\Leftrightarrow\frac{a}{b}\sqrt{a^2+b^2}\geq\frac{1}{\sqrt{2}}\cdot\frac{a(a+b)}{b}$$

$$\frac{a(a+b)}{b}-\frac{a^2-b^2}{a+3b}=(a+b)\left(\frac{a}{b}-\frac{a-b}{a+3b}\right)=\cdots=\frac{(a+b)^3}{b(a+3b)}>0$$

$$\Leftrightarrow\frac{a(a+b)}{b}>\frac{a^2-b^2}{a+3b}\Leftrightarrow\frac{a}{b}\sqrt{a^2+b^2}\geq\frac{1}{\sqrt{2}}\cdot\frac{a(a+b)}{b}>\frac{1}{\sqrt{2}}\cdot\left(\frac{a^2-b^2}{a+3b}\right)$$

$$Thus,\quad\sum_{cyc}\frac{a}{b}\sqrt{a^2+b^2}>\sum_{cyc}\frac{1}{\sqrt{2}}\cdot\left(\frac{a^2-b^2}{a+3b}\right)$$

$$\frac{a}{b}\sqrt{a^2+b^2}+\frac{b}{c}\sqrt{b^2+c^2}+\frac{c}{a}\sqrt{c^2+a^2}>\frac{1}{\sqrt{2}}\left(\frac{a^2-b^2}{a+3b}+\frac{b^2-c^2}{b+3c}+\frac{c^2-a^2}{c+3a}\right)$$