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In ΔABC the following relationship holds:

$$\frac{csc^5\left(\frac{A}{2}\right)}{a}+\frac{csc^5\left(\frac{B}{2}\right)}{b}+\frac{csc^5\left(\frac{C}{2}\right)}{c}\geq\frac{32\sqrt{3}}{R}$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{csc^5\left(\frac{A}{2}\right)}{a}+\frac{csc^5\left(\frac{B}{2}\right)}{b}+\frac{csc^5\left(\frac{C}{2}\right)}{c}\overset{Holder}{\geq}\frac{\left(csc\frac{A}{2}+csc\frac{B}{2}+csc\frac{C}{2}\right)^5}{3^{5-2}(a+b+c)}\overset{AM-GM}{\geq}$$

$$\geq\frac{\left(3\left(\prod_{cyc}csc\frac{A}{2}\right)^{\frac{1}{3}}\right)^5}{27.2s}=\frac{9\left(\prod\frac{1}{sin\frac{A}{2}}\right)^{\frac{5}{3}}}{2s}=\frac{9\left(\frac{4R}{r}\right)^{\frac{5}{3}}}{2s}\overset{Euler}{\geq}$$

$$\geq\frac{9.(8)^{\frac{5}{3}}}{2s}\overset{Mitrinovic}{\geq}\frac{9\cdot 32}{3\sqrt{3}R}=\frac{32\sqrt{3}}{R}$$

$$\textit{Equality holds for } a=b=c.$$