

# ROMANIAN MATHEMATICAL MAGAZINE

In  $\triangle ABC$  the following relationship holds:

$$\frac{1}{a^5} + \frac{1}{b^5} + \frac{1}{c^5} \geq \frac{1}{3\sqrt{3}R^5}$$

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*Solution by Daniel Sitaru-Romania*

$$\begin{aligned} \frac{1}{a^5} + \frac{1}{b^5} + \frac{1}{c^5} &= \frac{1^6}{a^5} + \frac{1^6}{b^5} + \frac{1^6}{c^5} \stackrel{RADON}{\geq} \\ &\geq \frac{(1+1+1)^6}{(a+b+c)^5} = \frac{3^6}{(2s)^5} \stackrel{MITRINOVICI}{\geq} \frac{3^6}{\left(2 \cdot \frac{3\sqrt{3}}{2} R\right)^5} = \frac{3^6}{3^5 \cdot 9\sqrt{3}R^5} = \frac{3}{9\sqrt{3}R^5} = \frac{1}{3\sqrt{3}R^5} \end{aligned}$$

Equality holds for  $a = b = c$ .