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In $\triangle ABC$ the following relationship holds:

$$\frac{1}{\sin^5 \frac{A}{2}} + \frac{1}{\sin^5 \frac{B}{2}} + \frac{1}{\sin^5 \frac{C}{2}} \geq 96$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Daniel Sitaru-Romania

$$\begin{aligned} \frac{1}{\sin^5 \frac{A}{2}} + \frac{1}{\sin^5 \frac{B}{2}} + \frac{1}{\sin^5 \frac{C}{2}} &= \frac{1^6}{\sin^5 \frac{A}{2}} + \frac{1^6}{\sin^5 \frac{B}{2}} + \frac{1^6}{\sin^5 \frac{C}{2}} \stackrel{RADON}{\geq} \\ &= \frac{(1+1+1)^6}{\left(\sin \frac{A}{2} + \sin \frac{B}{2} + \sin \frac{C}{2}\right)^5} \stackrel{JENSEN}{\geq} \frac{3^6}{\left(3 \sin \frac{\frac{A+B+C}{2}}{3}\right)^5} = \\ &= \frac{3^6}{\left(3 \sin \frac{A+B+C}{6}\right)^5} = \frac{3}{\sin^5 \frac{\pi}{6}} = \frac{3}{\frac{1}{32}} = 96 \end{aligned}$$

Equality holds for $A = B = C$.