ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{m_a}{r_a}+\frac{m_b}{r_b}+\frac{m_c}{r_c}\geq\frac{729r^3}{\left(m_a\sqrt{r_a}+m_b\sqrt{r_b}+m_c\sqrt{r_c}\right)^2}$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{m_a}{r_a}+\frac{m_b}{r_b}+\frac{m_c}{r_c}=$$

$$=\frac{m_ar_br_c+m_br_ar_c+m_cr_br_a}{r_ar_br_c}\geq\frac{729r^3}{\left(m_a\sqrt{r_a}+m_b\sqrt{r_b}+m_c\sqrt{r_c}\right)^2}$$

$$Let's\ prove\ that\ :$$

$$\frac{(m_ar_br_c+m_br_ar_c+m_cr_br_a)\left(m_a\sqrt{r_a}+m_b\sqrt{r_b}+m_c\sqrt{r_c}\right)^2}{r_ar_br_c}\geq 729r^3$$

$$\frac{(m_ar_br_c+m_br_ar_c+m_cr_br_a)\left(m_a\sqrt{r_a}+m_b\sqrt{r_b}+m_c\sqrt{r_c}\right)^2}{r_ar_br_c}\overset{AM-GM}{\geq}$$

$$\geq\frac{3\left((m_ar_br_c)(r_ar_br_c)^2\right)^{\frac{1}{3}}\left(3\left((m_ar_br_c)(r_ar_br_c)^{\frac{1}{2}}\right)^{\frac{1}{3}}\right)^2}{r_ar_br_c}=$$

$$=\frac{27\left((m_ar_br_c)(r_ar_br_c)\right)}{r_ar_br_c}=27(m_ar_br_c)\geq 27h_ah_bh_c\geq 27.27r^3=729r^3$$

$$Equality\ holds\ for\ a=b=c.$$