ROMANIAN MATHEMATICAL MAGAZINE

$$\text{If in } \Delta ABC,\ A = \frac{\pi}{7}, B = \frac{2\pi}{7}, C = \frac{4\pi}{7} \text{ then prove that:}$$

$$R^4\left(\frac{1}{a^4} + \frac{1}{b^4} + \frac{1}{c^4}\right) > \frac{27}{49}$$

Proposed by Tapas Das-India

Solution by Daniel Sitaru-Romania

$$R^4\left(\frac{1}{a^4} + \frac{1}{b^4} + \frac{1}{c^4}\right) = R^4\left(\frac{\left(\frac{1}{a^2}\right)^2}{1} + \frac{\left(\frac{1}{b^2}\right)^2}{1} + \frac{\left(\frac{1}{c^2}\right)^2}{1}\right) \overset{RADON}{\geq}$$

$$> R^4 \cdot \frac{\left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}\right)^2}{1 + 1 + 1} = R^4 \cdot \frac{\left(\frac{2}{R^2}\right)^2}{3} = \frac{4}{3} > \frac{27}{49}$$

THE HEPTAGONAL TRIANGLE REVISITED

DANIEL SITARU, CLAUDIA NĂNUŢI - ROMANIA

ABSTRACT. In this paper are proved the characteristic metric relationships in the heptagonal triangle.

We call heptagonal triangle the obtuse scalene triangle whose vertices coincide with the first, second and fourth vertices of a regular heptagon (from an arbitrary starting vertex).
Thus its sides coincide with one side and the adjacent shorter and longer diagonals of the regular heptagon.

....

In conclusion, in a heptagonal triangle with sides $a < b < c; \mu(A) = \frac{\pi}{7}$; $\mu(B) = \frac{2\pi}{7}; \mu(C) = \frac{4\pi}{7}$ the following relationship holds:

$$a^2 = c^2 - bc; b^2 = a^2 + ac; c^2 = b^2 + ba$$

$$\frac{1}{a} = \frac{1}{b} + \frac{1}{c};$$

$$h_a = h_b + h_c; h_a^2 + h_b^2 + h_c^2 = \frac{a^2 + b^2 + c^2}{2}$$

$$F = \frac{\sqrt{7}}{4} R^2;$$

$$a^2 + b^2 + c^2 = 7R^2; \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{2}{R^2}$$

□

References

[1] Romanian Mathematical Magazine - Interactive Journal, www.ssmrmh.ro