ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{16R}{r}-\frac{(sinA+sinB+sinC)^4}{4sin^2Asin^2Bsin^2C}\le 5$$

Proposed by Samir Cabiyev-Azerbaijan

Solution by Daniel Sitaru-Romania

$$\frac{16R}{r}-\frac{(sinA+sinB+sinC)^4}{4sin^2Asin^2Bsin^2C}=\frac{16R}{r}-\frac{\left(\frac{a+b+c}{2R}\right)^4}{4\left(\frac{abc}{8R^3}\right)^2}=$$

$$=\frac{16R}{r}-\frac{\left(\frac{2s}{2R}\right)^4}{4\left(\frac{4RF}{8R^3}\right)^2}=\frac{16R}{r}-\left(\frac{s}{R}\right)^4\cdot\frac{64R^6}{4\cdot 16R^2r^2s^2}=$$

$$=\frac{16R}{r}-\frac{s^4}{R^4}\cdot\frac{R^4}{r^2s^2}=\frac{16R}{r}-\frac{s^2}{r^2}\overset{GERRETSEN}{\underbrace{\le}}\frac{16R}{r}-\frac{16Rr-5r^2}{r^2}=$$

$$=\frac{16R}{r}-\frac{16R}{r}+5=5$$

Equality holds for $A=B=C$.