ROMANIAN MATHEMATICAL MAGAZINE

If in ΔABC $a \geq b$ *and* $x \in (0,1)$ *then prove that :*

$$\frac{3c(b+c-a)}{x^2+x+1}+\frac{2(a-b)(a-b-c)}{x+1}+(a-b)c>0$$

Proposed by Pavlos Trifon-Greece

Solution by Tapas Das-India

$$x \in (0,1) \text{ then } 1<x+1<2 \text{ and } x^2+x+1<1^2+1+1=3 \text{ so}$$
$$\frac{1}{x+1}<1 \;\&\; \frac{1}{x^2+x+1}>\frac{1}{3}$$

$$3c(b+c-a)>0 \text{ as in any } \Delta ABC\ b+c>a$$

$$2(a-b)(a-b-c)=-2(a-b)(b+c-a)<0$$
$$\text{as } a \geq b \;\&\; \text{ in any } \Delta ABC\ b+c>a \text{ and } (a-b)c>0 \text{ as } a \geq b$$

$$\frac{3c(b+c-a)}{x^2+x+1}+\frac{2(a-b)(a-b-c)}{x+1}+(a-b)c>0$$

$$\frac{1}{3}3c(b+c-a)+2(a-b)(a-b-c)+(a-b)c>0$$

$$c(b+c-a)+2(a-b)(a-b-c)+(a-b)c>0$$

$$c\big(c-(a-b)\big)+2(a-b)\big((a-b)-c\big)+(a-b)c>0$$

$$c^2-c(a-b)+2(a-b)^2-2c(a-b)+(a-b)c>0$$

$$c^2-2c(a-b)+2(a-b)^2>0$$

$$\big((a-b)-c\big)^2+(a-b)^2>0 \text{ true}$$