ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{a^2+b^2}{r_c}+\frac{b^2+c^2}{r_a}+\frac{c^2+a^2}{r_b}\geq 24r$$

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Solution by Daniel Sitaru – Romania

$$\frac{a^2+b^2}{r_c}+\frac{b^2+c^2}{r_a}+\frac{c^2+a^2}{r_b}=\sum_{cyc}\frac{a^2+b^2}{r_c}\overset{AM-GM}{\geq}\sum_{cyc}\frac{2ab}{r_c}=2\sum_{cyc}\frac{ab}{r_c}\overset{AM-GM}{\geq}$$

$$=2\cdot 3\sqrt[3]{\frac{(abc)^2}{r_ar_br_c}}=6\sqrt[3]{\frac{16r^2F^2}{Fs}}=6\cdot 2\sqrt[3]{\frac{2R^2F}{s}}=12\sqrt[3]{\frac{2R^2rs}{s}}=12\sqrt[3]{2R^2r}\geq$$

$$\geq 12\sqrt[3]{2\cdot 4r^2\cdot r}=12\cdot 2r=24r$$

Equality holds for $a=b=c$.