ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{(a+b)^2}{h_c}+\frac{(b+c)^2}{h_a}+\frac{(c+a)^2}{h_b}\geq 48r$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\frac{(a+b)^2}{h_c}+\frac{(b+c)^2}{h_a}+\frac{(c+a)^2}{h_b}=\sum_{cyc}\frac{(a+b)^2}{h_c}\geq$$

$$\overset{AM-GM}{\geq} 3\sqrt[3]{\frac{\left((a+b)(b+c)(c+a)\right)^2}{h_a h_b h_c}}\overset{CESARO}{\geq} 3\sqrt[3]{\frac{(8abc)^2}{h_a h_b h_c}}=$$

$$=3\sqrt[3]{\frac{64\cdot(4RF)^2}{\frac{2F}{a}\cdot\frac{2F}{b}\cdot\frac{2F}{c}}}=\frac{3}{2F}\cdot 4\sqrt[3]{(4RF)^2\cdot abc}=$$

$$=\frac{6}{F}\sqrt[3]{(4RF)^3}=\frac{6}{F}\cdot 4RF=24R\overset{EULER}{\geq}24\cdot 2r=47r$$

Equality holds for $a=b=c$.