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In ΔABC the following relationship holds:

$$(ab)^{\sin C}\cdot(bc)^{\sin A}\cdot(ca)^{\sin B}\le\left(3R^2\right)^{\frac{3\sqrt{3}}{2}}$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$(ab)^{\sin C}\cdot(bc)^{\sin A}\cdot(ca)^{\sin B}\overset{WEIGHTED\ AM-GM}{\le}$$

$$\le\left(\frac{ab\sin C+bc\sin A+ca\sin B}{\sin A+\sin B+\sin C}\right)^{\sin A+\sin B+\sin C}=\left(\frac{2F+2F+2F}{\frac{s}{R}}\right)^{\frac{s}{R}}\overset{MITRINOVIC}{\le}$$

$$\le\left(\frac{6F}{\frac{s}{R}}\right)^{\frac{1}{R}\cdot\frac{3\sqrt{3}}{2}R}=\left(\frac{6FR}{s}\right)^{\frac{3\sqrt{3}}{2}}=\left(\frac{6rsR}{s}\right)^{\frac{3\sqrt{3}}{2}}=(6rR)^{\frac{3\sqrt{3}}{2}}\le\left(6\cdot\frac{R}{2}\cdot R\right)^{\frac{3\sqrt{3}}{2}}=\left(3R^2\right)^{\frac{3\sqrt{3}}{2}}$$

Equality holds for $a=b=c$.