ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$r_a r_b + r_b r_c + r_c r_a \geq h_a h_b + h_b h_c + h_c h_a$$

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Solution by Daniel Sitaru – Romania

$$\sum_{cyc} r_a r_b \geq \sum_{cyc} h_a h_b, \qquad \sum_{cyc} \frac{F}{s-a} \cdot \frac{F}{s-b} \geq \sum_{cyc} \frac{2F}{a} \cdot \frac{2F}{b}$$

$$\sum_{cyc} \frac{1}{(s-a)(s-b)} \geq 4 \sum_{cyc} \frac{1}{ab}, \quad \frac{s-c+s-b+s-a}{(s-a)(s-b)(s-c)} \geq 4 \cdot \frac{a+b+c}{abc}$$

$$\frac{s}{(s-a)(s-b)(s-c)} \geq 4 \cdot \frac{2s}{4RF}, \quad \frac{s^2}{s(s-a)(s-b)(s-c)} \geq \frac{2s}{R \cdot rs}$$

$$\frac{s^2}{F^2} \geq \frac{2}{Rr}, \qquad \frac{s^2}{r^2 s^2} \geq \frac{2}{Rr}$$

$$\frac{1}{r^2} \geq \frac{2}{Rr} \Leftrightarrow R \geq 2r \quad \text{(Euler)}$$

Equality holds for $a = b = c$.