ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{h_a}{b}+\frac{h_b}{c}+\frac{h_c}{a}\le\frac{3\sqrt{3}}{4}\cdot\frac{R}{r}$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\frac{h_a}{b}+\frac{h_b}{c}+\frac{h_c}{a}=\sum_{cyc}\frac{h_a}{b}=\sum_{cyc}\frac{\frac{2F}{a}}{b}=2F\sum_{cyc}\frac{1}{ab}=2F\sum_{cyc}\frac{c}{abc}=$$

$$=\frac{2F}{abc}\sum_{cyc}c=\frac{2F}{4RF}(a+b+c)=\frac{1}{2R}\cdot 2s=\frac{s}{r}\overset{EULER}{\le}\frac{s}{2r}\overset{MITRINOVIC}{\le}$$

$$\le\frac{3\sqrt{3}}{2}R\cdot\frac{1}{2r}=\frac{3\sqrt{3}}{4}\cdot\frac{R}{r}$$

Equality holds for $a=b=c$.