ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{a+b}{\sin A}+\frac{b+c}{\sin B}+\frac{c+a}{\sin C}\geq 12R$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\frac{a+b}{\sin A}+\frac{b+c}{\sin B}+\frac{c+a}{\sin C}=\sum_{cyc}\frac{a+b}{\sin A}=\sum_{cyc}\frac{a+b}{\frac{a}{2R}}=2R\sum_{cyc}\frac{a+b}{a}\overset{AM-GM}{\geq}$$

$$\geq 2R\cdot 3\sqrt[3]{\frac{(a+b)(b+c)(c+a)}{abc}}\overset{CESARO}{\geq}6R\cdot\sqrt[3]{\frac{8abc}{abc}}=6R\sqrt[3]{8}=6R\cdot 2=12R$$

Equality holds for $a=b=c$.