ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{a}{\sin^3 B}+\frac{b}{\sin^3 C}+\frac{c}{\sin^3 A}\geq 8R$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\frac{a}{\sin^3 B}+\frac{b}{\sin^3 C}+\frac{c}{\sin^3 A}=\sum_{cyc}\frac{a}{\sin^3 B}\overset{AM-GM}{\geq}3\cdot\sqrt[3]{\frac{abc}{\sin^3 A\sin^3 B\sin^3 C}}=$$

$$=3\sqrt[3]{\frac{abc}{\left(\frac{abc}{2R}\right)^3}}=3\sqrt[3]{\frac{(8R^3)^3}{(abc)^2}}=\frac{24R^3}{\sqrt[3]{(abc)^2}}=\frac{24R^3}{\sqrt[3]{16R^2r^2s^2}}\overset{EULER}{\geq}$$

$$\geq\frac{24R^3}{\sqrt[3]{16R^2\cdot\frac{R^2}{4}s^2}}\overset{MITRINOVIC}{\geq}\frac{24R^3}{\sqrt[3]{4R^4\cdot\frac{27R^2}{4}}}=\frac{24R^3}{3R^2}=8R$$

Equality holds for $a=b=c$.