In ΔABC the following relationship holds:

$$\frac{a+b}{(\sin A+\sin B)^3}+\frac{b+c}{(\sin B+\sin C)^3}+\frac{c+a}{(\sin C+\sin A)^3}\geq 2R$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru – Romania

$$\frac{a+b}{(\sin A+\sin B)^3}+\frac{b+c}{(\sin B+\sin C)^3}+\frac{c+a}{(\sin C+\sin A)^3}=$$

$$=\sum_{cyc}\frac{a+b}{(\sin A+\sin B)^3}=\sum_{cyc}\frac{a+b}{\left(\frac{a}{2R}+\frac{b}{2R}\right)^3}=8R^3\sum_{cyc}\frac{a+b}{(a+b)^3}=8R^3\sum_{cyc}\frac{1}{(a+b)^2}\overset{RADON}{\geq}$$

$$\geq 8R^3\cdot\frac{(1+1+1)^3}{(a+b+b+c+c+a)^2}=\frac{8R^3\cdot 27}{4\cdot 4s^2}=\frac{27R^3}{2s^2}\overset{MITRINOVICI}{\geq}\frac{27R^3}{2\cdot\left(\frac{3\sqrt{3}}{2}R\right)^2}=$$

$$=\frac{27R^3}{2\cdot\frac{27R^2}{4}}=2R$$

Equality holds for $a=b=c$.