ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{a+b}{(\sin A+\sin B)^2}+\frac{b+c}{(\sin B+\sin C)^2}+\frac{c+a}{(\sin C+\sin A)^2}\ge 2\sqrt{3}R$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\frac{a+b}{(\sin A+\sin B)^2}+\frac{b+c}{(\sin B+\sin C)^2}+\frac{c+a}{(\sin C+\sin A)^2}=$$

$$=\sum_{cyc}\frac{a+b}{(\sin A+\sin B)^2}=\sum_{cyc}\frac{a+b}{\left(\frac{a}{2R}+\frac{b}{2R}\right)^2}=4R^2\sum_{cyc}\frac{a+b}{(a+b)^2}=$$

$$=4R^2\cdot\sum_{cyc}\frac{1^2}{a+b}\overset{BERGSTROM}{\ge}4R^2\cdot\frac{(1+1+1)^2}{a+b+b+c+c+a}=$$

$$=4R^2\cdot\frac{9}{4s}=\frac{9R^2}{s}\overset{MITRINOVIC}{\ge}\frac{9R^2}{\frac{3\sqrt{3}}{2}R}=\frac{3R\cdot 2}{\sqrt{3}}=2\sqrt{3}R$$

Equality holds for $a=b=c$.