ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$(r_a)^{a^2}.(r_b)^{b^2}.(r_c)^{c^2} \geq (3r)^{36r^2}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$(r_a)^{a^2}.(r_b)^{b^2}.(r_c)^{c^2} \overset{Weighted\ \ GM-HM}{\geq} \left(\frac{a^2+b^2+c^2}{\frac{a^2}{r_a}+\frac{b^2}{r_b}+\frac{c^2}{r_c}}\right)^{a^2+b^2+c^2} \overset{Chebyshev}{\geq}$$

$$\geq \left(\frac{(a^2+b^2+c^2)}{\frac{1}{3}\left((a^2+b^2+c^2)\left(\frac{1}{r_a}+\frac{1}{r_b}+\frac{1}{r_c}\right)\right)}\right)^{a^2+b^2+c^2} = (3r)^{a^2+b^2+c^2} \overset{Neuberg}{\geq} (3r)^{36r^2}$$

$$Equality\ \ holds\ \ if\ \ a=b=c.$$