ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$(m_a)^{b^2+c^2}(m_b)^{a^2+c^2}(m_c)^{b^2+a^2} \geq (3r)^{72r^2}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$(m_a)^{b^2+c^2}(m_b)^{a^2+c^2}(m_c)^{b^2+a^2} \overset{Weighted}{\geq} \left(\frac{2(a^2+b^2+c^2)}{\frac{a^2+b^2}{m_c}+\frac{c^2+b^2}{m_a}+\frac{a^2+c^2}{m_b}}\right)^{2\sum a^2} \overset{Chebyshev}{\geq}$$

$$\geq \left(\frac{2(a^2+b^2+c^2)}{\frac{1}{3}\left(2(a^2+b^2+c^2)\left(\frac{1}{m_a}+\frac{1}{m_b}+\frac{1}{m_c}\right)\right)}\right)^{2\sum a^2} \geq \left(\frac{3}{\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c}}\right)^{2\sum a^2} \overset{Neuberg}{\geq}$$

$$\geq \left(\frac{3}{\frac{1}{r}}\right)^{72r^2} = (3r)^{72r^2}$$

Equality holds if $a=b=c$.