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In $\triangle ABC$ the following relationship holds:

$$a^2 \cos(A) + b^2 \cos(B) + c^2 \cos(C) \leq \frac{9R^2}{2}$$

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In $\triangle ABC$ wlog $a \leq b \leq c$, $\cos(A) \geq \cos(B) \geq \cos(C)$

$$\begin{aligned} a^2 \cdot \cos(A) + b^2 \cdot \cos(B) + c^2 \cdot \cos(C) &\stackrel{\text{Chebyshev}}{\leq} \\ &\leq \frac{1}{3}(a^2 + b^2 + c^2)(\cos(A) + \cos(B) + \cos(C)) \stackrel{\text{Leibniz}}{\leq} \\ &\leq \frac{1}{3} \cdot 9R^2(\cos(A) + \cos(B) + \cos(C)) \stackrel{\text{Jensen}}{\leq} 3R^2 \cdot \frac{3}{2} = \frac{9R^2}{2} \end{aligned}$$

Equality holds iff $a = b = c$.