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In $\triangle ABC$ the following relationship holds:

$$a \tan \left(\frac{A}{2} \right) + b \tan \left(\frac{B}{2} \right) + c \tan \left(\frac{C}{2} \right) \geq 3R$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\begin{aligned} a \tan \left(\frac{A}{2} \right) + b \tan \left(\frac{B}{2} \right) + c \tan \left(\frac{C}{2} \right) &= a \cdot \frac{r_a}{s} + b \cdot \frac{r_b}{s} + c \cdot \frac{r_c}{s} = \\ &= \frac{ar_a + br_b + cr_c}{s} = \frac{2s(2R - r)}{s} = 4R - 2r \stackrel{\text{Euler}}{\geq} 3R \end{aligned}$$

Equality holds for $a = b = c$.