ROMANIAN MATHEMATICAL MAGAZINE

**In $\Delta ABC$ the following relationship holds:**

$$\cos\frac{A}{2}+\cos\frac{B}{2}+\cos\frac{C}{2}\geq 3\sqrt{3}\cdot\frac{r}{R}$$

*Proposed by Nguyen Hung Cuong – Vietnam*

*Solution by Daniel Sitaru – Romania*

$$\cos\frac{A}{2}+\cos\frac{B}{2}+\cos\frac{C}{2}\overset{AM-GM}{\geq} 3\sqrt[3]{\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}=$$

$$=3\sqrt[3]{\sqrt{\frac{s(s-a)\cdot s(s-b)\cdot s(s-c)}{bc\cdot ac\cdot ab}}}=$$

$$=3\sqrt[6]{\frac{s^3(s-a)(s-b)(s-c)}{(abc)^2}}=3\sqrt[6]{\frac{s^2F^2}{(abc)^2}}=3\sqrt[3]{\frac{sF}{abc}}=3\sqrt[3]{\frac{sF}{4RF}}=3\sqrt[3]{\frac{s}{4R}}$$

**Remains to prove:**

$$3\sqrt[3]{\frac{s}{4R}}\geq 3\sqrt{3}\cdot\frac{r}{R}\Leftrightarrow\frac{s}{4R}\geq 3\sqrt{3}\cdot\frac{r^3}{R^3}$$

$$sR^3\geq 12\sqrt{3}Rr^3\Leftrightarrow sR^2\geq 12\sqrt{3}r^3\quad\text{(to prove)}$$

$$sR^2\overset{EULER}{\geq} s\cdot(2r)^2\overset{MITRINOVIC}{\geq} 3\sqrt{3}r\cdot 4r^2=12\sqrt{3}r^3$$

**Equality holds for $A=B=C$.**