ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{a+b}{h_a}+\frac{b+c}{h_b}+\frac{c+a}{h_c}\geq 4\sqrt{3}$$

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Solution by Daniel Sitaru – Romania

$$\frac{a+b}{h_a}+\frac{b+c}{h_b}+\frac{c+a}{h_c}=\sum_{cyc}\frac{a+b}{h_a}\overset{AM-GM}{\geq}$$

$$\geq 3\sqrt[3]{\frac{(a+b)(b+c)(c+a)}{h_ah_bh_c}}\overset{CESARO}{\geq} 3\sqrt[3]{\frac{8abc}{\frac{2F}{a}\cdot\frac{2F}{b}\cdot\frac{2F}{c}}}=$$

$$=3\sqrt[3]{\frac{(abc)^2}{F^3}}=3\sqrt[3]{\frac{16R^2F^2}{F^3}}=3\sqrt[3]{\frac{16R^2}{F}}\overset{MITRINOVIC}{\geq} 3\sqrt[3]{\frac{16\cdot\frac{4}{27}s^2}{rs}}=3\cdot\frac{4}{3}\sqrt[3]{\frac{s}{r}}\geq$$

$$\overset{MITRINOVIC}{\geq} 4\sqrt[3]{\frac{3\sqrt{3}r}{r}}=4\sqrt[3]{\left(\sqrt{3}\right)^3}=4\sqrt{3}$$

Equality holds for $a=b=c$.