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In ΔABC the following relationship holds:

$$\frac{AI\cdot BI\cdot CI}{abc}\le\frac{1}{3\sqrt{3}}$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\frac{AI\cdot BI\cdot CI}{abc}=\frac{\frac{r}{\sin\frac{A}{2}}\cdot\frac{r}{\sin\frac{B}{2}}\cdot\frac{r}{\sin\frac{C}{2}}}{abc}=\frac{r^3}{abc\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}=$$

$$=\frac{r^3}{abc\sqrt{\frac{(s-b)(s-c)(s-a)(s-c)(s-a)(s-b)}{bc\cdot ac\cdot ab}}}=$$

$$=\frac{r^3}{abc\cdot\frac{(s-a)(s-b)(s-c)}{abc}}=\frac{r^3s}{s(s-a)(s-b)(s-c)}=$$

$$=\frac{r^3s}{F^2}=\frac{r^3s}{r^2s^2}=\frac{r}{s}\overset{MITRINOVIC}{\le}\frac{r}{3\sqrt{3}r}=\frac{1}{3\sqrt{3}}$$

Equality holds for $a=b=c$.